



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

which equals the square of the *difference* of the two roots, or

$$(2n+1 + 2\sqrt{\frac{n(n+1)}{2}})^2$$

Illustration.—From the series of triangular square numbers,  $1^2$ ,  $6^2$ ,  $35^2$ ,  $204^2$ ,  $1189^2$ , etc., take 6 and 35.  $35 - 6 = 29$ ;  $35 + 6 = 41 = 20 + 21$ ;  $20^2 + 21^2 = 29^2$ .

This problem and problems No. 45, (Vol. III., No. 5, page 153), and No. 36, of Diophantine Analysis, are very closely related.

Also solved by the *PROPOSER*.

52. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics in Columbian University, Washington, D. C.

Prove that a "magic square" of nine integral elements, whose rows, columns, and diagonals have a constant sum, is only possible when this sum is a multiple of three.

I. Solution by M. W. HASKELL, M. A., Ph. D., Associate Professor of Mathematics, University of California, Berkeley, California.

Let the magic square be 
$$\begin{array}{|c|c|c|} \hline a & b & c \\ \hline d & e & f \\ \hline g & h & k \\ \hline \end{array}$$
 and let  $S$  be the constant sum.

Then  $S = a + b + c = d + e + f = g + h + k = a + d + g = b + e + h = c + f + k = a + e + k = c + e + g$ .

Adding these all together, we have  $8S = 3a + 2b + 3c + 2d + 4e + 2f + 3g + 2h + 3k = 3(a + c + g + k) + 2(b + e + h) + 2(d + f)$ . But the last two quantities in parenthesis are each  $= S$ . Hence  $4S = 3(a + c + g + k)$ , and  $S$  is a multiple of 3.

II. Solution by — (Paper Unsigned.)

Suppose the numbers occupying the magic square to be  $a, b, c, d, e, f, g, h, k$ . Now  $a + e + k = b + e + h = c + e + g = S$ .

$\therefore a + k \equiv k \pmod{3}$ ,  $b + h \equiv k \pmod{3}$ ,  $c + g \equiv k \pmod{3}$ , where  $S - e \equiv k \pmod{3}$ .

Adding the congruences,  $(a + b + c) + (g + h + k) \equiv 0 \pmod{3}$ . Or, since  $(a + b + c) + (g + h + k) \equiv 0 \pmod{3}$ ,  $2S \equiv 0 \pmod{3}$ .

Multiply by 2, and divide by 3, and the result is  $S \equiv 0$ . Q. E. D.

III. Solution by W. H. CARTER, Professor of Mathematics, Centenary College of Louisiana, Jackson, Louisiana.

Let the rows of the "square" be  $a, b, c$ ;  $x, y, z$ ; and  $l, m, n$ , and let the constant sum be  $k$ . We have to show that  $k/3$  is integral. We have  $a + y + n = k$ ;  $b + y + m = k$ ;  $l + y + c = k$ . Add, and we have  $(a + b + c) + (l + m + n) + 3y = 3k$ , that is,  $2k + 3y = 3k$ .

$\therefore 3y = k$ .  $\therefore y = k/3$ . But  $y$  is integral.  $\therefore k/3$  is integral.

Also solved by M. A. GRUBER and G. B. M. ZERR.